

Universal Landau Pole

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Abstract

Our understanding of quantum gravity suggests that at the Planck scale the usual geometry loses its meaning. If so, the quest for grand unification in a large non-abelian group naturally endowed with the property of asymptotic freedom may also lose its motivation. Instead we propose an unification of all fundamental interactions at the Planck scale in the form of a *Universal Landau Pole* (ULP), at which all gauge couplings diverge. The Higgs quartic coupling also diverges while the Yukawa couplings vanish. The unification is achieved with the addition of fermions with vector gauge couplings coming in multiplets and with hypercharges identical to those of the Standard Model. The presence of these particles also prevents the Higgs quartic coupling from becoming negative, thus avoiding the instability (or metastability) of the SM vacuum.

Under the renormalization group flow the coupling constants of the three fundamental gauge interactions behave quite differently [1]. While the couplings of the non-abelian interactions, weak and strong, constantly diminish with as the energy increases, the coupling of the abelian interaction grows, and eventually diverges, a phenomenon usually referred as *Landau pole* [2].

This results from a direct extrapolation of physics at present energies. The existence of new particles, or in general new physics, alter this behaviour. For sometime it was thought that the three interactions coincided at a single scale, and this was interpreted to signal the presence of a non-abelian grand unified group. Present accurate data show that this triple coincidence does not happen in the absence of new physics. The weak and hypercharge couplings are equal at a scale of the order of 10^{12} GeV, hypercharge and strong at around 10^{14} GeV and the two non-abelian couplings meet around 10^{17} GeV.

Continuing the flow of the three couplings beyond the previous scales would give rise to a "weak" force actually stronger than the "strong" one, and the abelian coupling overtaking both of them. If one continues the running of the hypercharge coupling it diverges at the finite, albeit extremely large, scale of 10^{53} GeV. Even though the calculations are done perturbatively at a finite loop order and the value of the scale where the pole occurs is therefore not to be trusted numerically (since the perturbative expansion will have broken before), the qualitative behaviour will however remain: for a non asymptotic free theory at some energy there will be a pole.

Also relevant is the behaviour under the renormalization flow of the quartic Higgs coupling λ and of the Yukawa couplings of the top y_t , which is the largest of the fermion couplings and therefore dominates at high energy. They both decrease, but while y_t remains positive, λ becomes negative in the presence of a relatively light Higgs with mass ~ 125 GeV [3]. This signals an instability, or at least a metastable phase, of the theory.

The idea of the unification of the forces is very appealing. A grand unified group guarantees the presence of asymptotic freedom and consequently the ability to describe particles and fields at arbitrarily small distances. However, we know that in nature there is also gravity and before the Planck scale, around $m_p = 10^{19}$ GeV, the onset of quantum gravity will certainly alter in a substantial way the picture. Models of emergent gravity (see e.g. [4]) indicate that there may be a 'smallest distance' below which the very notion of length may not exist. In any case, dramatic quantum gravity effects —perhaps a string theory—

are likely to manifest themselves at around the Planck scale and it is not obvious at all why one should expect quantum field theory to remain perturbatively valid beyond the Planck scale. Then, the philosophical necessity for asymptotic freedom at the most fundamental scale weakens considerably.

In this letter we want to put forward another type of unification. Namely, the proposal that all coupling constants, as well as the Yukawa couplings and the quartic Higgs coupling have a singularity at an energy of the order of the Planck mass. This common singularity, which we term *Universal Landau Pole* (ULP) may be interpreted as signaling the onset of a phase transition to radically new physics. The nature of the “high energy” trans-planckian phase is not known, there would probably be some sort of quantum space-time and hypothesis abound. The existence of a common singularity might hint that this new phase could be weakly coupled, but in a completely different set of variables. We assume that singularity at the transition shows up as a pole in *all* gauge couplings and the quartic coupling, and a zero for the Yukawa coupling. In the following we will see that the model presented here also solves the potentially disastrous instability of the Higgs potential [5].

The recent Large Hadron Collider measurement [3] of the Higgs mass around 125 ± 1 GeV together with the absence of new physics indicate that the quartic coupling of the Higgs self-interaction may become negative at an energy as low as 10^8 GeV [6] suggesting an instability of the theory. In terms of the effective potential this is tantamount to a negative quadric term, and therefore the potential is not bounded from below.

In the scenario we propose, the presence of new particles solves the stability problem, and at the same time generates the ULP. In order to achieve this the new physics must hasten the running of the abelian coupling towards the pole, lowering it from $\sim 10^{53}$ to about 10^{19} GeV, modify the running of the quartic coupling, and avoid the appearance of problems in known physics. In the following we show one possible model where we achieve the objectives listed above. The aim of this exercise is to show a reasonable possibility where a ULP with the required properties manifests itself.

The possibility that a Landau pole may be present at the Planck mass is not totally new. The authors of [7] used this hypothesis to set bounds for the number and masses of quarks and leptons. Later the possibility of an unification at strong coupling has been studied in the context of GUT and SUSY, see e.g. [8, 9], and references therein for a review. We follow a different line: ULP is related with physics at the Planck scale and we do not introduce a

new gauge group nor SUSY, and no new gauge or Higgs fields. Another interesting proposal, which has some similarity with ours, relies on the possible existence of a non-gaussian UV fixed point. This conjecture arises from the possibility of gravity being an asymptotically safe theory [10] and assuming that all other interactions unify at the same non-perturbative fixed point (see recent discussion in [11]) and, remarkably enough, this conjecture may have interesting implications on the quartic coupling as well. This latter proposal does not assume a radical new theory for gravity –just an hypothetical non-perturbative completion of the usual theory.

From now on we will adhere to the minimal hypothesis of the existence of a ULP. In this letter we will present our calculations at the one loop level. Present theoretical knowledge would enable us to perform the calculation up to three loops [12]. We will present the full analysis of the two-loops case in a forthcoming work [13].

At one loop the running of the couplings is given by simple equations

$$\frac{dg_i(t)}{dt} = \beta_i(t), \quad \beta_i \equiv \frac{1}{16\pi^2} g_i^3 b_i, \quad t \equiv \log \frac{\mu}{GeV}. \quad (1)$$

Where $i = 1, 2, 3$ represent the U(1), SU(2) and SU(3) couplings respectively. The presence of new particles will alter this running. In the following we will consider that the new particles come with the same quantum numbers as the known ones. To differentiate the new from the old particles we will call them “quarkons” and “leptos”. To avoid problems with anomalies, and to avoid the introduction of new Higgs-like particles, all new particles are vector Dirac particles, but they maintain the representations of the known gauge groups. Baryon and lepton charges for new fermions are conserved separately. Quarkons are SU(3) triplets and leptons are SU(3) singlets. They both come in two kinds: SU(2) doublets, which we will call L-quarkons and L-leptos, with the hypercharges of left quarks and left leptons respectively; and SU(2) singlets, R quarkons, with the hypercharge of right handed quarks, and R-leptos carrying the hypercharge of right electrons. However, R-leptos which are singlets for all SM groups (the equivalent of right neutrinos) could be in principle present, but since they do not contribute to the running of the gauge couplings we can ignore them. We emphasize that all quarkons and leptons are *vector-like* particles. Thus the Higgs boson Lagrangian has the form of SM and therefore the one-loop RG flow of quartic coupling and of Yukawa couplings are fully controlled by behavior of gauge couplings.

It may seem that there is a large degree of arbitrariness in this construction and indeed

there is some freedom. In fact, while we do not make any claims to uniqueness, it is not so easy to construct models with all the necessary ingredients and it is nontrivial that a solution exists at all with the constraints of using only the representations and hypercharges of the standard model, apart from the fact that fermions are vector-like. Heavier particles are necessary to avoid the couplings to be asymptotically free, but their presence affects at least two couplings, therefore it is not obvious that the boundary condition of a single universal pole can be satisfied. Other possibilities we have explored contemplate a hypothetical restoration of the left-right symmetry in the weak interactions, but we shall not consider this here and postpone these extensions to subsequent work.

We will assume that the various particles have masses such that they contribute only when a particular threshold of energy is reached. Thus the behaviour of the running couplings is piecewise continuous, with the coefficients of the equation depending on the scale.

For the $U(1)$ gauge coupling g_1 , the constant b_1 is given by:

$$b_1 = \frac{41}{6} + \frac{2}{3}N_{\text{L-leptos}} + \frac{4}{3}N_{\text{R-leptos}} + \frac{2}{9}N_{\text{L-quarkon}} + \frac{4}{3}N_{\text{R-quarkon}}. \quad (2)$$

For the $SU(2)$ gauge coupling we have:

$$b_2 = -\frac{10}{3} + \frac{2}{3}N_{\text{L-leptos}} + 2N_{\text{L-quarkon}}. \quad (3)$$

For the $SU(3)$ gauge coupling

$$b_3 = -7 + \frac{4}{3}(N_{\text{L-quarkon}} + N_{\text{R-quarkon}}). \quad (4)$$

The integers N in these formulas refer to the number of quarkon and leptos multiplets contributing to beta functions.

It is immediate from (1) that the flow equation for

$$\frac{1}{\alpha_i} \equiv \frac{4\pi}{g_i^2} \quad (5)$$

are linear. Since the coefficients are piecewise constant, and change at the energies representing the scale at which the new particles, it is possible to do a systematic search. We have imposed as boundary condition of the differential equation that $1/\alpha_i = 0$ at the Planck scale m_p . In any case the model cannot be trusted at energies approaching m_p for more than one

reason. The perturbative approach will have broken down, not to speak of the one loop approximation, and moreover gravitational effects could not be ignored. Our setting a precise boundary condition giving a common pole at a particular scale is therefore just expedient to describe a common pole that the present theoretical tools cannot properly describe.

The other low energy boundary conditions are given by the experimental values: $\alpha_s = 0.1184$, $g_1 = 0.358729$, $g_2 = 0.648382$, $g_3 = 1.16471$, $y = 0.937982$, $\lambda = 0.125769$ for $M_H = 124\text{GeV}$ at the scale of the top mass $\mu = M_t = 172.9\text{GeV}$. These values are insensitive to M_H in the range $124 - 126\text{ GeV}$. Since the equations are linear (at one loop), the results presented here are quite “robust” for slight changes in the boundary condition, both in the low energy and pole regions.

We found that the following choice of quarkons and leptos gives good results. We consider that there are four identical “generations” each of them consisting of one multiplet of particles with the following scales:

- At the scale of $5.0 \cdot 10^3\text{ GeV}$ the L-quarkons ($N_{\mathbf{L-quarkon}} = 4$).
- At $3.7 \cdot 10^7\text{ GeV}$ the R-quarkons ($N_{\mathbf{R-quarkon}} = 4$).
- At $2.6 \cdot 10^{14}$ the L and R-leptos ($N_{\mathbf{L-leptos}} = N_{\mathbf{R-leptos}} = 4$).

In Fig. 1 we show the running of the gauge coupling (the initial running is actually made

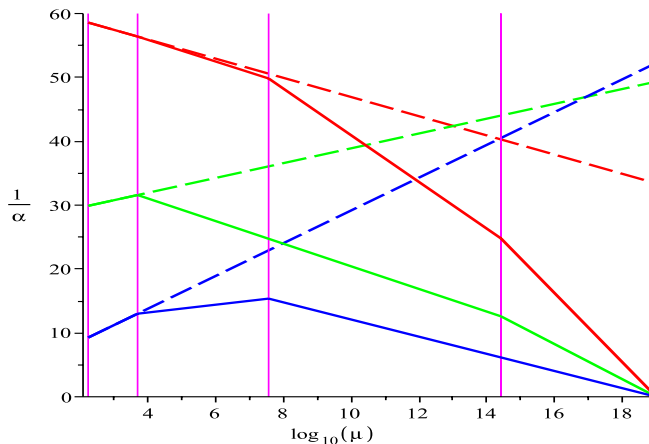


FIG. 1. The running of α_i , the inverse of the gauge couplings. The dotted lines are the runnings in the absence of quarkons and leptos. The α_i are in descending order as i increases.

with the two-loop equation). One can see that the hierarchy of the couplings is respected,

the strong force remains stronger than the weak. The scale at which there is the appearance of the new particles is larger than the experimental bounds on the presence of new fermions, but not too much. This scenario shows that the ULP may exist with new physics at energies within reach. Other solutions are possible and we will comment on them in [13], although the qualitative features of these alternative options are similar to the one presented here.

The running of the gauge couplings affects the other couplings we considered. As far the top Yukawa coupling is concerned the equation is

$$\beta_y^{(1)} = \frac{1}{(4\pi)^2} y \left(-\frac{9}{4} g_2^2 - \frac{17}{12} g_1^2 - 8 g_3^2 + \frac{9}{2} y^2 \right). \quad (6)$$

Solving the differential equation we obtain Fig. 2. The coupling is undistinguishable from

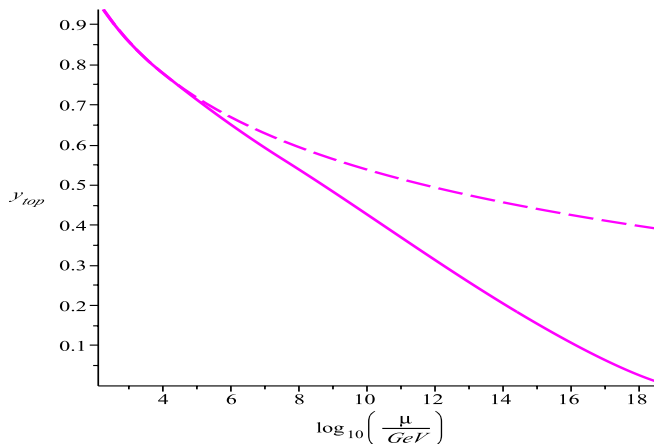


FIG. 2. The running of y_t . Again the dotted line is the SM case.

the one of the standard model for energies up to 10^6 GeV, and vanishes at the ULP.

The quartic coupling equation is

$$\begin{aligned} \beta_\lambda^{(1)} = \frac{1}{16\pi^2} \left(24 \lambda^2 - 6 y^4 + \frac{3}{4} g_2^4 + \frac{3}{8} (g_2^2 + g_1^2)^2 \right. \\ \left. + (-9 g_2^2 - 3 g_1^2 + 12 y^2) \lambda \right). \end{aligned} \quad (7)$$

Its solution, given the couplings we calculated earlier, is shown in Fig. 3. We see that the quartic coupling for our choice of new particles comes close to vanish, but never actually becomes negative. At the ULP the coupling diverges. The potential term will therefore dominate over the other terms.

The scenario described in this letter may give hints to aspects of the trans-planckian phase. In such a phase the kinetic term becomes negligible and the propagator “freezes”,

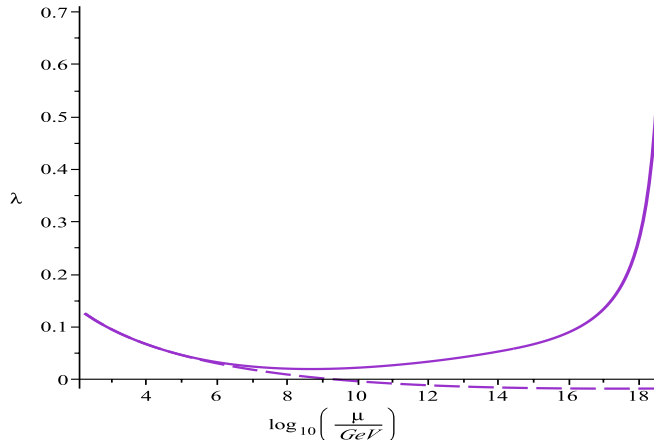


FIG. 3. *The running of quartic coupling of the Higgs field. The dotted line shows the instability that the standard model develops in the presence of a Higgs mass of 124-126 GeV.*

and the Higgs (which might be composite) decouples. This suggests that gauge bosons may possibly be effective (probably composite too) degrees of freedom. Gravity will play a dominant role, but the absence of propagating degrees of freedom suggests a “geometry” without points, with space-time possibly described by a noncommutative geometry [14] and/or replaced by a pregeometric entity, such as the spin foam and spin networks of quantum gravity [15]. In proposals such as [4], even gravity is described entirely in terms of fermions, which may describe the whole physical world if an scenario such as the one proposed here is realized.

It could well be the case that the onset of gravity corrections renders the ULP we advocate in this paper non-singular. Indeed gravity being non-renormalizable will require higher-dimensional operators with more derivatives to render the theory finite. In particular, we expect dimension six kinetic terms like

$$\frac{\gamma}{2M_P^2} \text{tr} (D_\mu W^{\mu\nu} D_\mu W_\nu^\mu) + \dots \quad (8)$$

This would correspond to a renormalization of the gauge coupling induced by gravity of the form

$$\frac{1}{g^2(p^2)} \simeq \beta_0 \log \frac{m_P^2}{p^2} + \gamma \frac{p^2}{m_P^2}. \quad (9)$$

As shown in [11] gravitational corrections may drive the ULP towards a new fixed point.

The renormalization flow of the various constants, especially in view of the new data coming from LHC, may be an essential tool for the understanding of physics at the gravitational

frontier.

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